





















## 7 CONCLUSION AND FUTURE WORK

In this paper, we proposed SpaceNE, introducing subspaces to the field of community network embedding. To the best of our knowledge, this work is the first attempt of introducing subspaces into network representation learning. Specifically, we design elaborate objectives preserving proximity between pairwise nodes, across communities, along with constraints on subspace dimension which was then approximated by a differentiable one, leading to efficient optimization of our model. Empirically, we verify SpaceNE in a variety of datasets and applications. Extensive experimental results demonstrate the advantages of SpaceNE, especially on link prediction, hierarchical community preservation.

Here we focus on the hierarchical network embedding by using subspace theory. Nevertheless, the theory of subspace is largely overlooked in the field of network representation learning. For future work, one intriguing direction is utilizing subspace theory to deal with the heterogeneity of complex networks. Also, in real-life scenes, such as neighborhood-based recommendation, when searching for the nearest neighbor of an item, the search engine only needs to search in the lower-dimensional subspace, which can improve the efficiency greatly.

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